SCAT 1st Latin-American Summer School
January 9th, 2007

Introduction to Turbulence and its Numerical Simulation

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Objectives of course (I & II):

- Give an introduction to turbulent flow characteristics and physics,
- Provide a first, basic understanding of standard turbulence models used in Computational Fluid Dynamics (CFD)
- Provide basic understanding of Direct Numerical (DNS) and Large Eddy Simulation (LES)
- Illustrate state-of-the-art subgrid model for LES and applications.

Prerequisites:

- Basic Fluid Mechanics,
- Tensors and Index Notation

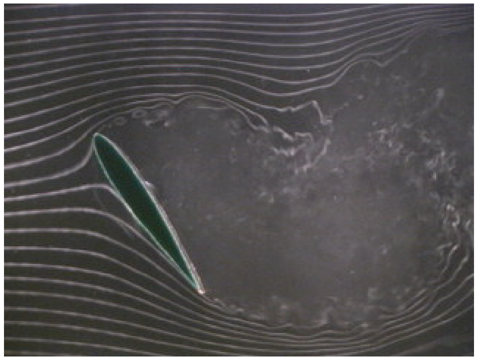
Outline (I):

- Overview of turbulent flow characteristics,
- Reynolds decomposition,
- Turbulence physics and energy cascade,
- Turbulence modeling for CFD:
  Eddy-viscosity and k-ε model
- Filtering, Large Eddy Simulation (LES)
- Direct Numerical Simulation (DNS)

Outline (II):

- Smagorinsky model and coefficient calibration,
- Non-universality and problems in complex flows,
- Dynamic model and applications

Turbulent flows:



From: Multimedia Fluid Mechanics, Cambridge Univ. Press

- **multiscale,**
- **mixing,**
- **dissipative,**
- **chaotic,**
- **vortical**
- **well-defined statistics,**
- **important in practice**

Turbulent flows:

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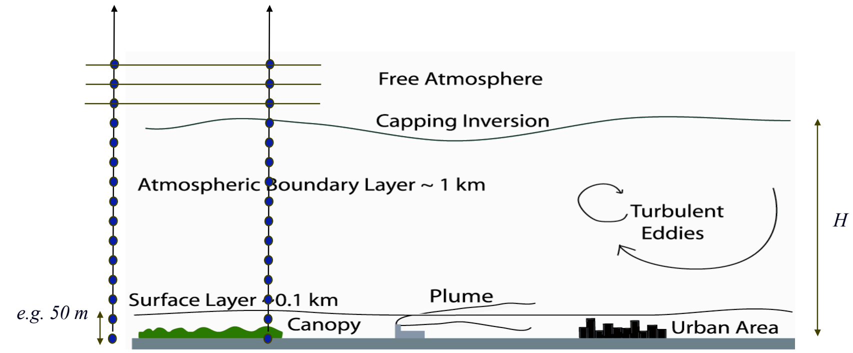
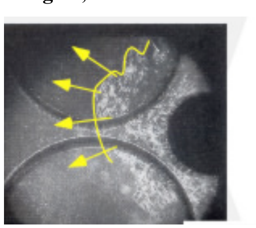
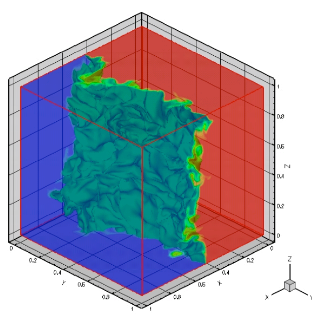
Turbulence in atmospheric boundary layer



Turbulence in reacting flows:

Premixed flame in I.C. engine, combustion

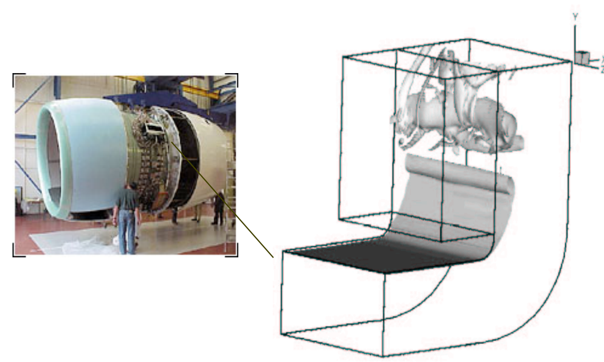




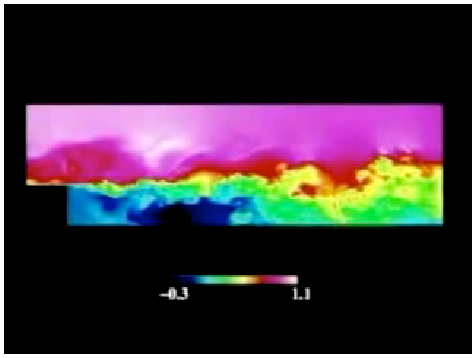
Numerical simulation of flame propagation in decaying isotropic turbulence

Turbulence in aerospace systems:

LES of flow in thrust-reversers
Blin, Hadjadi & Vervisch (2002)
J. of Turbulence.



Turbulence in thermofluid equipment:



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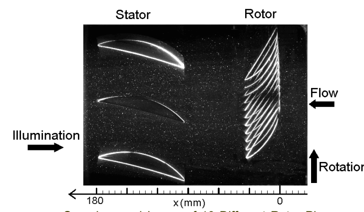
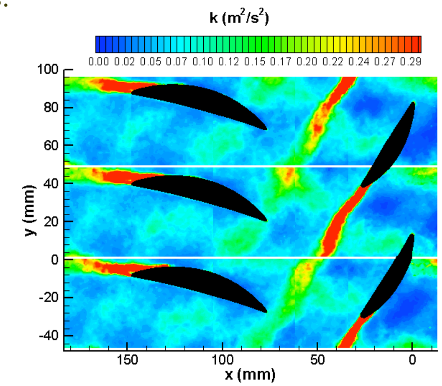
Turbulence in turbomachines:

Optical velocity field measurements in index-matched axial pump
Phase-averaged turbulent kinetic energy distribution
(Uzol, Katz & Meneveau, J. Turbomach. 2003)

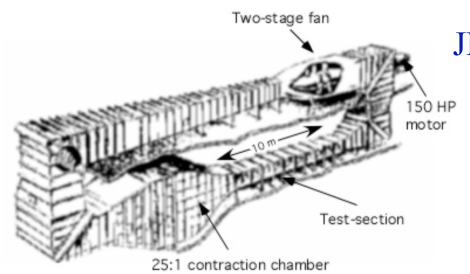
Lattice of wakes:



Superimposed Image of 10 Different Rotor Phases at 500 rpm and at the Mid-span location. x=0 is at rotor leading edge. The stage length is L_s = 203 mm.

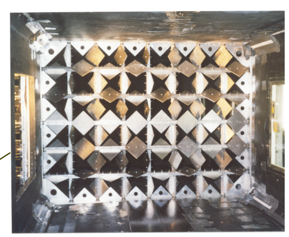


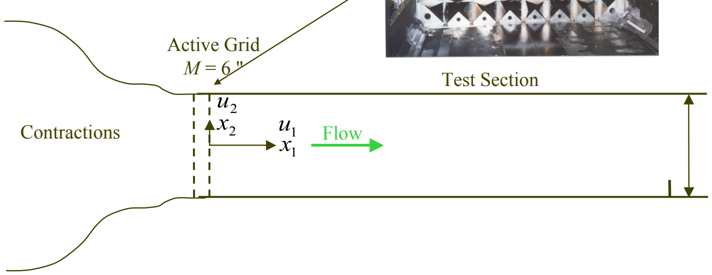
Simplest turbulence: Isotropic decaying turbulence



JHU Corrsin wind tunnel

Active Grid
$M = 6$ "





Physical quantities describing fluid flow

- Density field
- Velocity vector field
- Pressure field
- Temperature field (or internal energy, or enthalpy etc..)

Physical laws governing fluid flow

- Conservation of mass
- Newton's second law
- First law of thermodynamics
- Equation of state
- Some constraints in closure relations from second law of TD

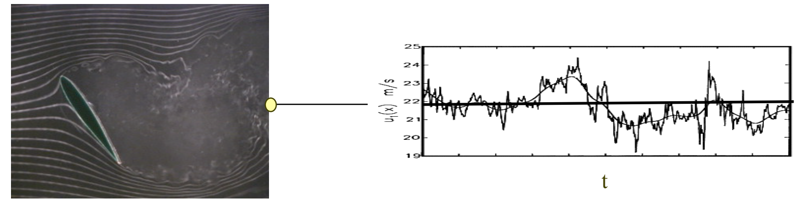
Navier Stokes equations for a Newtonian, incompressible fluid

Navier-Stokes equations, incompressible, Newtonian

$$
\begin{cases}
\dfrac{\partial u_j}{\partial x_j} = 0 \\
\underbrace{\dfrac{\partial u_j}{\partial t} + \dfrac{\partial u_k u_j}{\partial x_k} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x_j} + \nu \nabla^2 u_j + g_j}_{a_j = \frac{F_j}{m}}
\end{cases}
$$

Turbulence: Reynolds decomposition

$$
\begin{cases}
\dfrac{\partial u_j}{\partial x_j} = 0 \\
\dfrac{\partial u_j}{\partial t} + \dfrac{\partial u_k u_j}{\partial x_k} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x_j} + \nu \nabla^2 u_j + g_j
\end{cases}
$$



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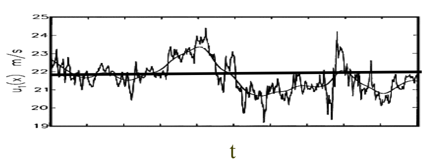
Reynolds' equations:



$$
\begin{cases}
\dfrac{\partial \bar{u}_j}{\partial x_j} = 0 \\
\dfrac{\partial \bar{u}_j}{\partial t} + \dfrac{\partial \bar{u}_j \bar{u}_k}{\partial x_k} = -\dfrac{1}{\rho}\dfrac{\partial \bar{p}}{\partial x_j} + \nu \nabla^2 \bar{u}_j + g_j - \dfrac{\partial}{\partial x_k}\left(\overline{u_j u_k} - \bar{u}_j \bar{u}_k\right)
\end{cases}
$$

- **Reynolds stress**
- **Energy cascade**
- **Spectral energy tensor**
- **Isotropic turbulence**
- **Kolmogorov spectrum (1941)**

• Kinematic Reynolds stress (minus):

$$
\begin{cases}
\dfrac{\partial \bar{u}_j}{\partial x_j} = 0 \\[2ex]
\dfrac{\partial \bar{u}_j}{\partial t} + \dfrac{\partial \bar{u}_j \bar{u}_k}{\partial x_k} = -\dfrac{1}{\rho}\dfrac{\partial \bar{p}}{\partial x_j} + \nu \nabla^2 \bar{u}_j + g_j - \dfrac{\partial}{\partial x_k}\underbrace{\left(\overline{u_j u_k} - \bar{u}_j \bar{u}_k\right)}
\end{cases}
$$

$$\sigma^R_{jk} = \overline{u_j u_k} - \bar{u}_j \bar{u}_k$$

Written as velocity co-variance tensor:

$$\sigma^R_{jk} = \overline{(\bar{u}_j + u'_j)(\bar{u}_k + u'_k)} - \bar{u}_j \bar{u}_k = \bar{\bar{u}}_j \bar{\bar{u}}_k + \cancelto{0}{\overline{\bar{u}_j u'_k}} + \cancelto{0}{\overline{\bar{u}_k u'_j}} + \overline{u'_j u'_k} - \bar{u}_j \bar{u}_k = \overline{u'_j u'_k}$$

0

• Kinematic Reynolds stress (minus):

$$
\begin{cases}
\dfrac{\partial \bar{u}_j}{\partial x_j} = 0 \\[2ex]
\dfrac{\partial \bar{u}_j}{\partial t} + \dfrac{\partial \bar{u}_j \bar{u}_k}{\partial x_k} = -\dfrac{1}{\rho}\dfrac{\partial \bar{p}}{\partial x_j} + \nu \nabla^2 \bar{u}_j + g_j - \dfrac{\partial \sigma^R_{jk}}{\partial x_k}
\end{cases}
$$

$$\sigma^R_{jk} \equiv \overline{u'_j u'_k}$$

• Deviatoric (anisotropic part): $\tau^R_{jk} \equiv \sigma^R_{jk} - \frac{1}{3}\sigma^R_{mm}\delta_{jk}$

$$\frac{\partial \bar{u}_j}{\partial t} + \frac{\partial \bar{u}_j \bar{u}_k}{\partial x_k} = -\frac{1}{\rho}\frac{\partial \overline{p*}}{\partial x_j} + \nu \nabla^2 \bar{u}_j + g_j - \frac{\partial \tau^R_{jk}}{\partial x_k}$$

• Kinematic Reynolds stress (minus):

$$
\begin{cases}
\dfrac{\partial \bar{u}_j}{\partial x_j} = 0 \\[2ex]
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\end{cases}
$$

Unknowns: mean velocity and pressure field

$\bar{u}_1(x_1, x_2, x_3, t)$,
$\bar{u}_2(x_1, x_2, x_3, t)$,
$\bar{u}_3(x_1, x_2, x_3, t)$,
$\bar{p}(x_1, x_2, x_3, t)$

Closure required for Reynolds stress tensor: express stress in terms of mean velocity field…

$$\tau^R_{jk} = func(\bar{\mathbf{u}})$$

• Spectral representation of co-variance tensor:

$$\overline{u'_j u'_k} = \frac{1}{(2\pi)^3}\iiint \Phi_{jk}(k_1, k_2, k_3)\, d^3\mathbf{k}$$

$\Phi_{jk}(k_1, k_2, k_3)$: Spectral tensor of turbulence

(how much energy there is in each wave vector **k**)

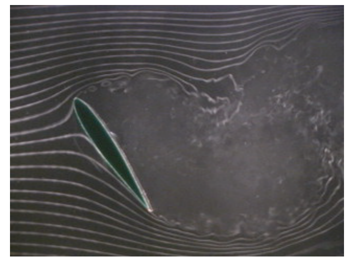
In **homogeneous isotropic turbulence** (simplest case, with no preferred directions) the spectral tensor function of a vector can be expressed based on a single scalar function of magnitude of wavenumber, *E(k):*

$$\Phi_{jk}(\mathbf{k}) = \frac{1}{4\pi k^2}\left(\delta_{jk} - \frac{k_j k_k}{k^2}\right) E(k)$$

• **What is the dependence of energy density *E(k)* with wavenumber?**

We now discuss the energy cascade and Kolmogorov theory of turbulence

• **Turbulence Physics: the energy cascade**
(Richardson 1922, Kolmogorov 1941)



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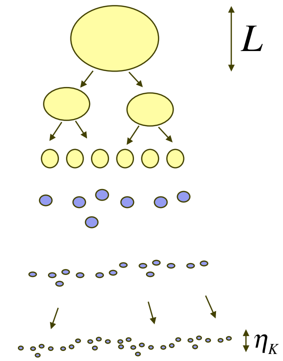


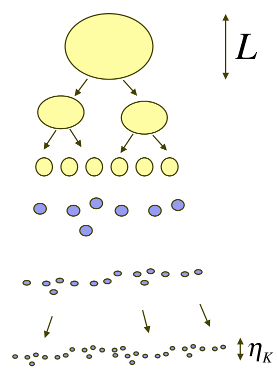
• **Turbulence Physics: the energy cascade**
(Richardson 1922, Kolmogorov 1941)

Big whorls have little whorls,
which feed on their velocity,
and little whorls have lesser whorls,
and so on to viscosity (in the molecular sense)

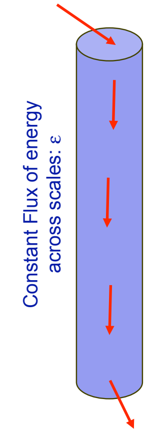
• **Turbulence Physics: the energy cascade**
(Richardson 1922, Kolmogorov 1941)

$$u' \equiv \sqrt{\frac{1}{3}\overline{u_i' u_i'}}$$

$$\frac{u'^2}{Time} \sim \frac{u'^2}{L/u'} \sim \frac{u'^3}{L}$$



Injection of kinetic energy into turbulence (from mean flow)



$$\varepsilon \sim \frac{u'^3}{L}$$

$$\varepsilon \sim \frac{u'(r_1)^3}{r_1}$$

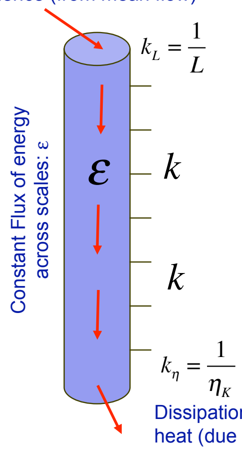
$$\varepsilon \sim \frac{u'(r_2)^3}{r_2}$$

$$\varepsilon = 2\nu \overline{\frac{\partial u_i'}{\partial x_j}\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right)}$$

Dissipation of kinetic energy into heat (due to molecular friction)

- **Turbulence Physics: the energy cascade**
  (Richardson 1922, Kolmogorov 1941)

Injection of kinetic energy into turbulence (from mean flow)



Dissipation of kinetic energy into heat (due to molecular friction)

$$E = f(k, \varepsilon)$$

Dimensional Analysis (Pi-theorem: 3-2=1):

$$\frac{E\, k^{5/3}}{\varepsilon^{2/3}} = const$$

$$E(k) = c_K \varepsilon^{2/3} k^{-5/3}$$

Solid experimental support for K-41:

Solid line is equivalent of a 3D radial spectrum equal to

$$E(k) = c_K \varepsilon^{2/3} k^{-5/3}, \quad c_K \sim 1.6$$

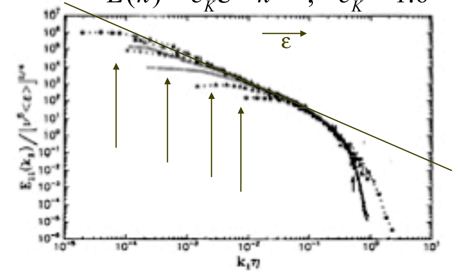


FIG. 5. Energy spectrum of the calibrated and filtered velocity signal from the turbulent wake and the grid turbulence (upper and lower solid lines, respectively), in Kolmogorov units. The arrow indicates the highest wave number for which the data can be trusted, since for $k_1\eta > 0.5$ the spectrum is influenced by filtering and probe size. Symbols: compilation of some representative data from other experiments. Squares: grid turbulence $R_\lambda = 37$ (Comte-Bellot and Corrsin, 1971). Triangles: grid turbulence $R_\lambda = 72$ (Comte-Bellot and Corrsin, 1971). Plus: cylinder wake $R_\lambda = 308$ (Uberoi and Freymuth, 1969). Rhombs: grid turbulence $R_\lambda = 540$ (Kistler and Vrebalovich, 1966). Stars: round jet $R_\lambda = 780$ (Gibson, 1963). Circles: boundary layer $R_\lambda = 1450$ (recent data of Veeravalli and Saddoughi, 1991).

J. O'Neil and C. Meneveau 163

Supports (approximately) the notion that ε is the only relevant physical scale in the inertial range,
+ L at large scales
+ ν at small scales

Back to Closure problem for Reynolds equations:

$$\frac{\partial \bar{u}_j}{\partial t} + \frac{\partial \bar{u}_j \bar{u}_k}{\partial x_k} = -\frac{1}{\rho}\frac{\partial \bar{p}^*}{\partial x_j} + \nu \nabla^2 \bar{u}_j + g_j - \frac{\partial \tau^R_{jk}}{\partial x_k}$$

$$\tau^R_{jk} = func(\bar{\mathbf{u}})$$

In analogy with molecular friction:

$$\tau^R_{jk} = -\nu_T \left(\frac{\partial \bar{u}_j}{\partial x_k} + \frac{\partial \bar{u}_k}{\partial x_j} \right)$$

"Eddy-viscosity" $\nu_T \sim (velocity\ scale) \times (length\ scale)$

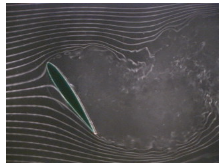
Kolmogorov:
Characterization of turbulence: minimum 2 variables

Turbulent kinetic energy: $K = \frac{1}{2}\overline{u_i' u_i'} = \frac{1}{2} u'^2$

$$\varepsilon \sim \frac{u'^3}{L} \iff \varepsilon \sim \frac{K^{3/2}}{L}$$

Need (K,L) or (ε,L) or $\boxed{(K,\varepsilon)}$

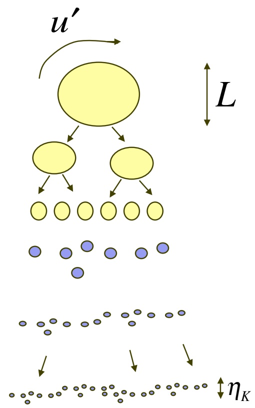
Eddy-viscosity scaling:



$$\nu_T \sim \underbrace{(velocity\ scale)}_{\sim u' \ \sim K^{1/2}} \times \underbrace{(length\ scale)}_{\sim L \ \sim \frac{K^{3/2}}{\varepsilon}}$$

$$\nu_T = C_\mu \frac{K^2}{\varepsilon}$$

$$\nu_T(\mathbf{x},t) = C_\mu \frac{K^2(\mathbf{x},t)}{\varepsilon(\mathbf{x},t)}$$



Transport equations for K(x,t) and ε(x,t)

Launder & Spalding (1972) - but earlier (1940s) Kolmogorov K-ω model

$$\frac{\partial K}{\partial t} + \bar{u}_k \frac{\partial K}{\partial x_k} = -\tau_{ij}^R \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\left(\frac{\nu_T}{\sigma_k} + \nu\right)\frac{\partial K}{\partial x_j}\right) - \varepsilon$$

$$\frac{\partial \varepsilon}{\partial t} + \bar{u}_k \frac{\partial \varepsilon}{\partial x_k} = C_{\varepsilon 1}\left(-\tau_{ij}^R \frac{\partial \bar{u}_i}{\partial x_j}\right)\frac{\varepsilon}{K} + \frac{\partial}{\partial x_j}\left(\left(\frac{\nu_T}{\sigma_\varepsilon} + \nu\right)\frac{\partial \varepsilon}{\partial x_j}\right) - C_{\varepsilon 2}\varepsilon\frac{\varepsilon}{K}$$

Empirical calibrations:
5 adjustable coefficients

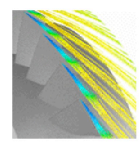
$$C_\mu = 0.09$$
$$C_{\varepsilon 1} = 1.44$$
$$C_{\varepsilon 2} = 1.92$$
$$\sigma_K = 1.0$$
$$\sigma_\varepsilon = 1.3$$

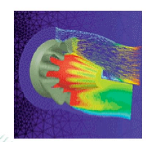
K-ε model for turbulence mean flow predictions:

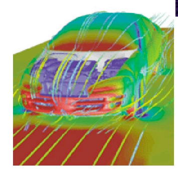
$$\begin{cases} \dfrac{\partial \bar{u}_j}{\partial x_j} = 0 \\ \dfrac{\partial \bar{u}_j}{\partial t} + \dfrac{\partial \bar{u}_j \bar{u}_k}{\partial x_k} = -\dfrac{1}{\rho}\dfrac{\partial \bar{p}}{\partial x_j} + g_j + \dfrac{\partial}{\partial x_k}\left[\left(\nu_T + \nu\right)\left(\dfrac{\partial \bar{u}_j}{\partial x_k} + \dfrac{\partial \bar{u}_k}{\partial x_j}\right)\right] \\ \dfrac{\partial K}{\partial t} + \bar{u}_k \dfrac{\partial K}{\partial x_k} = -\tau_{ij}^R \dfrac{\partial \bar{u}_i}{\partial x_j} + \dfrac{\partial}{\partial x_j}\left(\left(\dfrac{\nu_T}{\sigma_k} + \nu\right)\dfrac{\partial K}{\partial x_j}\right) - \varepsilon \\ \dfrac{\partial \varepsilon}{\partial t} + \bar{u}_k \dfrac{\partial \varepsilon}{\partial x_k} = C_{\varepsilon 1}\left(-\tau_{ij}^R \dfrac{\partial \bar{u}_i}{\partial x_j}\right)\dfrac{\varepsilon}{K} + \dfrac{\partial}{\partial x_j}\left(\left(\dfrac{\nu_T}{\sigma_\varepsilon} + \nu\right)\dfrac{\partial \varepsilon}{\partial x_j}\right) - C_{\varepsilon 2}\varepsilon\dfrac{\varepsilon}{K} \\ \text{+ boundary (\& initial) conditions} \end{cases}$$

Commercial CFD,
Steady RANS -> Standard

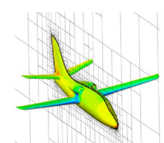
FLUENT™

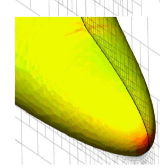






CFDRC™





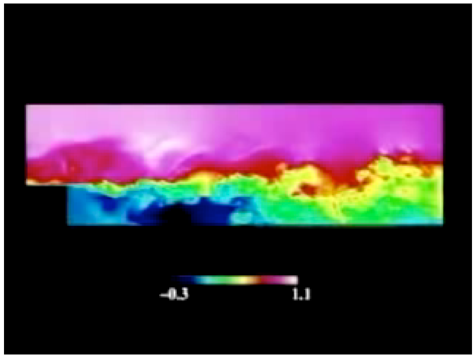
Applications:

- Aerospace (external)
- Chemical (mixing)
- Propulsion (internal flow)
- Biomedical
- HVAC
- Pollution, dispersion
- Etc...

Direct Numerical Simulation:

N-S equations:

$$\frac{\partial u_j}{\partial t} + \frac{\partial u_k u_j}{\partial x_k} = -\frac{\partial p}{\partial x_j} + \nu \nabla^2 u_j + g_j \qquad \frac{\partial u_j}{\partial x_j} = 0$$



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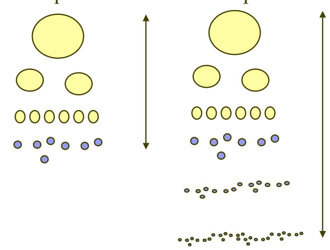
Moderate Re ($\sim 10^3$), DNS possible

High Re ($\sim 10^7$), DNS impossible



Regions of large vorticity in isotropic turbulence

Y. Kaneda and T. Ishihara

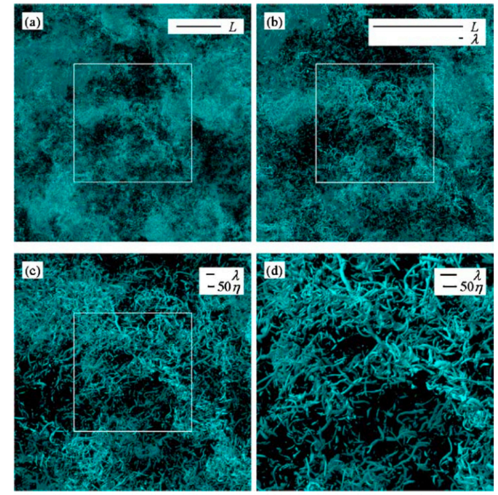
World-record DNS (Nagoya group in Japan):

On Earth Simulator (2003)

$4{,}096^3$ grid points

~ 2 Terabytes at each time-step

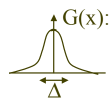
Thousands of time-steps

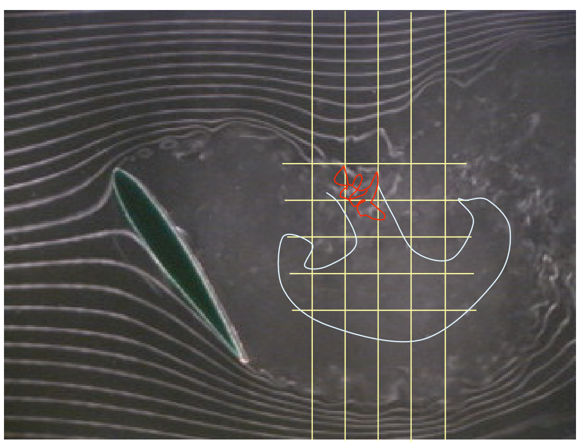


Source: Kaneda & Ishihara,
J. of Turbulence, 2006 © Taylor & Francis

Large-eddy-simulation (LES) and filtering:

$$\frac{\partial \tilde{u}_j}{\partial t} + \tilde{u}_k \frac{\partial \tilde{u}_j}{\partial x_k} = -\frac{\partial \tilde{p}}{\partial x_j} + \nu \nabla^2 \tilde{u}_j - \frac{\partial}{\partial x_k} \tau_{jk}$$

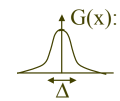




Large-eddy-simulation (LES) and filtering:

N-S equations:

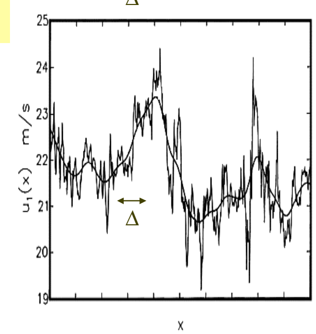
$$\frac{\partial u_j}{\partial t} + \frac{\partial u_k u_j}{\partial x_k} = -\frac{\partial p}{\partial x_j} + \nu \nabla^2 u_j \qquad \frac{\partial u_j}{\partial x_j} = 0$$



Filtered N-S equations:

$$\frac{\partial \tilde{u}_j}{\partial t} + \frac{\partial \widetilde{u_k u_j}}{\partial x_k} = -\frac{\partial \tilde{p}}{\partial x_j} + \nu \nabla^2 \tilde{u}_j$$

$$\frac{\partial \tilde{u}_j}{\partial t} + \tilde{u}_k \frac{\partial \tilde{u}_j}{\partial x_k} = -\frac{\partial \tilde{p}}{\partial x_j} + \nu \nabla^2 \tilde{u}_j - \frac{\partial}{\partial x_k} \tau_{jk}$$



where SGS stress tensor is:

$$\tau_{ij} = \widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j$$

Useful references:

- S. Pope: Turbulent Flow
  (Cambridge Univ. Press, 2000)

- J. Ferziger & M. Peric: Computational Methods for
  Fluid Dynamics (Springer, 1996)