Spiraling cracks in thin sheets
Final report for SCAT-alfa Project

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July 21, 2008

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Abstract

A wide kind of everyday-life industrial products come in a thin package that needs to be torn open by the user, and the opening is not always easy. We built a simple setup to study crack propagation in thin sheets coupled with large out-of-plane displacement : A cylindrical tool is inserted in a straight incision in a thin sheet, and is pushed against the sheet perpendicularly to that incision, eventually propagating a crack. When the blunt tool is continually pushed against the lip, we found that the crack follows a very robust spiraling path. Experiments may be interpreted in terms of "Spira Mirabilis" (logarithmic spiral). Starting with crack theory argument, we will show that the early behavior of the cut path follows a portion of a logathmic spiral, and that the path tends to another spiral with a different pitch as the crack adds more turns. Our crack experiment illustrates the fact that thin sheets mechanics is deeply connected to geometry, and finally spirals characteristics allow us to measure material crack properties of the thin layer used.

Abstract submitted and presented to APS March Meeting 2008.

1 Introduction

1.1 About fracture mechanics

Fracture mechanics is an opened field in continuous develop. In this field, despite the big numbers of achieves, there are still fundamental questions without any satisfactory answers.
A very first approach to fracture theory was performed by Leonardo Da Vinci. He showed that the strength of a metal wire varied inversely with the wire length.
Leonardo Da Vinci's result is hard to understand intuitively. It took over 200 year to find an explanations by Griffith [1], and with this, he founded the modern theory of fracture mechanics. It was also remarkable the work of Inglis [2], who solved, at the same time, the stress in a plate with an elliptical hole in it. He showed that a sharper corner concentrate the stress, and, as result of this, there are nucleation and fracture propagation.
Now on, fracture mechanics has become a very complex subject due to plasticity and nonlinearity nature of fractures due to plasticity.
In particular, a very important and fundamental question today in fracture mechanics is how to predict the crack path. Until now there are several criterions about this, however there is not a fundamental answer to this problem.
I will present some of the different criteria used to predict the crack path[3]:

criteria of the constrain of maximum opening: this criteria was the first to be proposed, the work of Erdogan and Sith established that the direction of the crack will be at some angle α gived by the condition of maximize the constrain $\sigma_{\theta\theta}$ at a fixed point $\mathbf{r}$, with the origin of the system of reference in the crack tip and $\mathbf{r}$ the distance to the crack tip.

criteria of the local symmetry: This idea, introduced by Goldstein and Salganik, says that the crack will follow the path which cancel the shear mode of fracture (Mode II). There is not a clear explanation about this idea, however an experiment performed by Erdogan and Sith [4] shows that this criteria predict approximately the behavior of the crack.

criteria of the maximum energy release rate: this criteria is very well known and has been used successfully to explain crack process in several configurations. It established that the crack will take the path which releases more energy.

In this work we will use the criterion of maximum energy release rate and Griffith criterion to model our results. Finally we are particularly interested in thin sheets because the fracture configuration became more interesting due to the richness in the geometry of thin sheets.

1.2 Previous experiment: oscillatory pattern in thin sheets

A schematics representation of the experiment is shown in the next Figure.

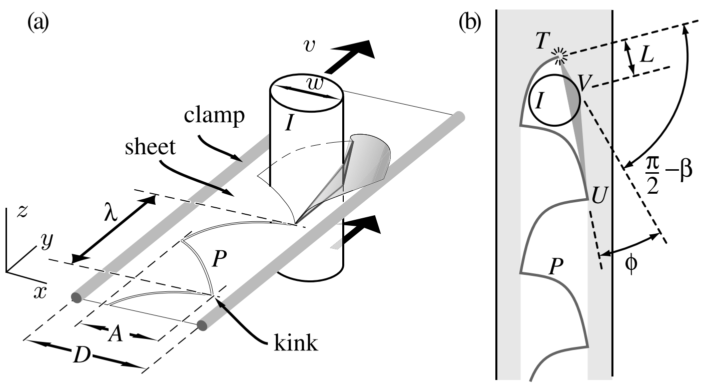

Figure 1: A thin sheet is clamped in its edges, a cylindrical tool goes through the thin sheet generating a crack. Depending of the diameter of the cylinder it became a very robust oscillatory path. **a)** A schematic view in perspective of the set up of the experiment. **b)** A view from above of the experiment.

In this setup a cylindrical tool is generating a crack in the thin sheet while it goes through it. Increasing the diameter of the tool the crack goes from an straight crack, as usually happen when a knife cut a thin sheet, to a very robust oscillatory pattern.
B. Roman et al. [5] gave an explanation for this pattern formation. In order to explain this lets think about one simple period of the crack path as we show in the figure bellow.

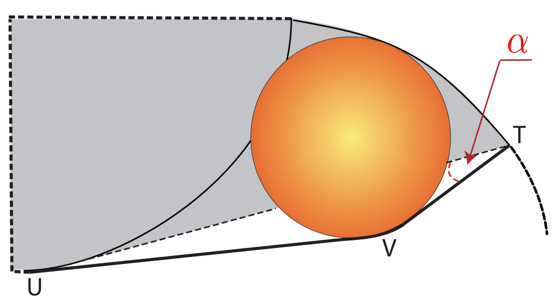

Figure 2: Diagram of the cylindrical tool pushing against the thin sheet in one single period of he crack oscillation.

The last figure represents the zone with the points T V U marked in Fig 1.b. In this figure, the orange circle represents the cylindrical tool, the dashed between points T and U is the lip when is unstressed, and the solid line from T through V to U is the lip of the sheet when it is stretched. When the tool goes to the right, at constant velocity, starts to push this line, stretching it and increasing the stress concentration at point T, the stress concentration increase in the point T. The angle α is the geometric parameter associated with the stress concentration. When α increases the stress at T increases. At some point, the stress, due to the increment of α, becomes enough to propagate the crack. This condition for the propagation is related with the Griffith criteria. One last thing to take in to account is that because the thin sheet is very easy to bend then the bending energy is neglected and crack propagation is due only to stretching energy. Finally we will assume that the crack goes perpendicular to the line T V.
The above outline assumptions allow to give a simple mechanism that explains our observations. In this work we are interested in study some question related with the validity of the assumptions used, such as the α limit value and the perpendicular propagation.

2 The Theory

We can think in a simple setup to analyze the crack propagation. A thin sheet is clamped at its edges, and an incision is made close to the center of the sample. This incision has, obviously, two lips. With a slim cylinder we push against one of this lips at some distance to the ends of the incision. This idea is shown in the following figure.

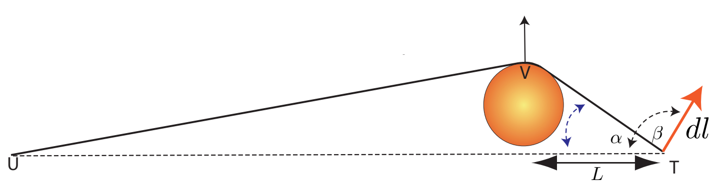

Figure 3: Diagram of the cylindrical tool pushing against on lip of the incision made in the thin sheet. Here we defined the parameters of the system, α, β, and L.

In the precedent diagram we show the variables that describe the stress concentration mechanism. Here the α angle is the angle between the line where the lip is unstressed (line UT) and the stressed lip (line VT), the angle β is the angle of propagation of the crack with an increment of dl. Finally the distance L is the distant of the tool to the closer end of the incision measured over the unstressed line (dash line in the figure).

2.1 The Model of Crack Propagation

As we said before we are neglecting out of plane bending energy, and we are taking in account only the stretching energy. This means that the energy have the form:

$$\mathcal{E} \propto E\varepsilon^2 Vol \tag{1}$$

In this equation, ε is the strain of the stretched lip, E is the young's modulus of the material, Vol is the volume of the zone stretched. Since $Vol \propto tL^2\alpha$ (t is the thickness, and L is the distance defined in the first figure of this subsection) and $\varepsilon \sim \alpha^2$, the energy has the form:

$$\mathcal{E} \propto EtL^2\varphi_{(\alpha)} \tag{2}$$

Where $\varphi_{(\alpha)}$ is a function that concentrate the alpha dependency. The following figure is used to determined the change in the quantities when the crack increase an infinitesimal distance dl.

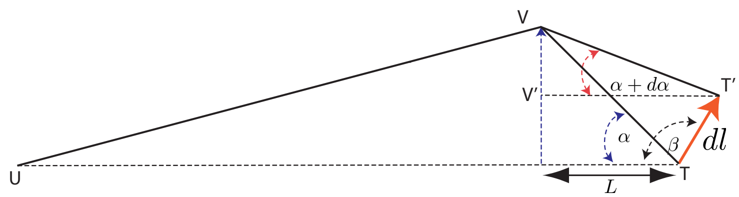

Figure 4: Diagram that show the change of the geometry when the crack advance an infinitesimal dl distance.

We are interested in the change of the energy when the crack advances a distance dl, so we are interested in the quantity, $d\mathcal{E}/dl$, called usually energy release rate. There is only one important parameter that change in the equation for the energy when the crack starts. It is α, so the change of the energy is proportional to $d\varphi_{(\alpha)}/dl$. A change in the variables yields:

$$\frac{d\mathcal{E}}{dl} \propto EtL^2 \frac{d\varphi_{(\alpha)}}{d\alpha}\frac{d\alpha}{dl} \tag{3}$$

To use this equation, we only need to get the term $d\alpha/dl$. This is implied by Fig. 4. From geometry (see Fig.4), we can write the following relation:

$$\tan(\alpha + d\alpha) = \frac{L\tan\alpha - dl\sin\alpha}{L - dl\cos\beta} \tag{4}$$

Expanding the left side of this equation, we find that:

$$\frac{\tan\alpha + \tan d\alpha}{1 - \tan\alpha\tan d\alpha} = \frac{L\tan\alpha - dl\sin\alpha}{L - dl\cos\beta} \tag{5}$$

Using infinitesimal quantities we find that:

$$\frac{d\alpha}{dl} = \frac{\cos^2\alpha}{L}(\tan\alpha\cos\beta - \sin\beta) \tag{6}$$

Here we need a criteria to predict the direction for the crack propagation. It makes sense that the crack will follow the path that allows the maximum release of energy. This means that the crack will choose the β angle for which the system gives more energy to the crack. Maximizing the quantity given by the last equation we obtain:

$$\frac{\partial}{\partial\beta}\frac{d\alpha}{dl} = 0 \tag{7}$$

therefore:

$$\tan\alpha = -\frac{1}{\tan\beta} \tag{8}$$

that is solved by the relation:

$$\beta = \alpha + \frac{\pi}{2} \tag{9}$$

It is evident from Fig. 4 that this result implies that the crack will propagate perpendicularly to the stressed lip.

2.2 The logarithmic equation

As we continuously push against the lip we will get a curved shape around the end of the initial incision which is not cracked. The idea at this point is to predict this shape. In the next Figure we show in polar coordinates the differential equation used to find the curve.

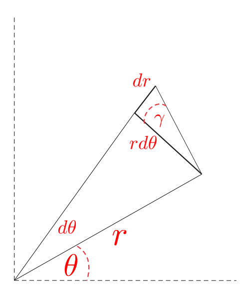

Figure 5: Geometry of polar coordinates.

A geometrical analysis in polar coordinates, as we can see from Fig. 5, gives us immediately the following relation:

$$\frac{dr}{r\,d\theta} = \tan\gamma \tag{10}$$

In particular, if the angle γ does not depend on θ, we can integrate this equation an obtain $r = e^{\tan(\gamma)\theta}$, which, indeed, is the equation for an "spira miravillis" or logarithmic spiral. Our experiment is shown in the next figure.

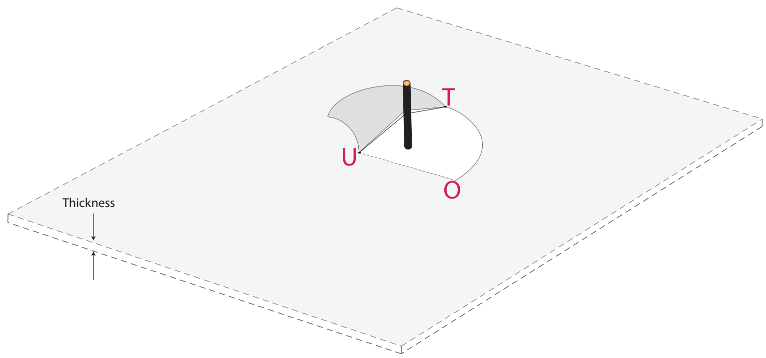

Figure 6: A thin sheet is clamped on it edges, an initial incision is made, line between point U and O in the Figure. With a blunt cylinder one of the lips of the incision is pushed continuously. The crack path until this point is the curve line between the points T and O in the Figure.The dark gray zone is the out of plane folding and is a non active zone in the process.

The angle β defined in section 2.1 is, in this experiment, the angle between an imaginary straight line joining points U and T and the tangent to the crack curve.
Note that this procedure is valid only until the crack have half turn around the point U in Fig. 6. What will happen after that point is that the crack will propagate around the point O in Fig. 6. This new axis of rotation will control the fracture propagation until the crack had turned a quarter of a turn more. After this second stage the propagation will continue to give a big final spiral built over itself.

3 The Experiment: A big Spiral

3.1 The Spiral construction

Until now we can justify the first and the second steps in the crack path. First one fracture grows with a direction β around the point U (see Fig. 6), after this crack has grown half of a turn. Second a spiral starts growing around the point O (see Fig. 6). When this second spiral turned one quarter of a turn the force cannot be applied between the crack tip and the point O, and a new stage start. It is hard to explain in words the process, Fig. 7 shows the different stages of the spiral construction.

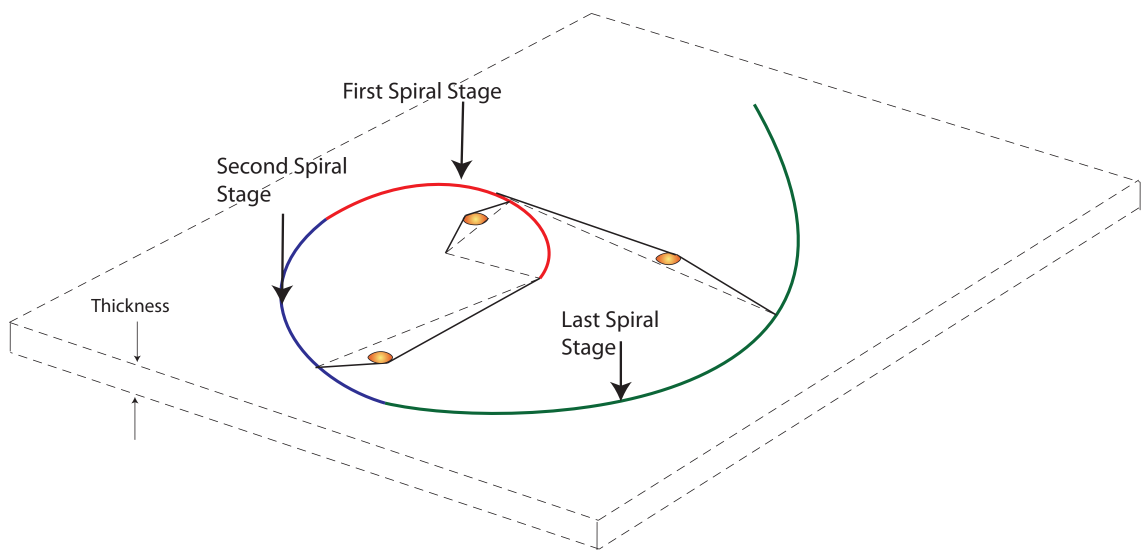

Figure 7: The Figure shows a diagram of different position of the tool at different moments in the fracture process. The different color-lines represent the different stages of the final crack path, the dashed lines are the unstressed lips at each moment, and the triangle formed in the shown position of the tool is the stressed lip.

In Fig. 7 the red and the blue lines are the first and the second step, respectively. With the green line we show what we mean with the final third step of the spiral construction. In this step the force will be applied in the longer line possible. In Fig. 7, this is the line between the green line and the outer point of the red line.

3.2 A real Spiral

A 30 [μm] thickness film was clamped in the edges in a 1[m] x 0.7[m] frame. A small 5 [mm] straight incision was made with a knife. With a cylindrical tool we pushed in one of the lips continuously. In the next figure we show a photograph of the observed spiral.

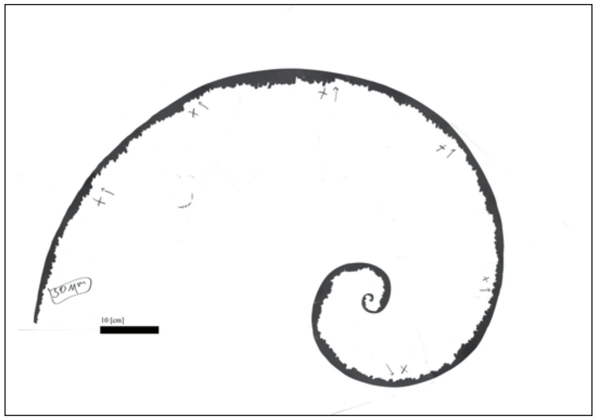

Figure 8: Picture of a real spiral. This picture was taken with a camera at maximal resolution. The black zone, painted in the spiral, allow us to identify the edge of the spiral and indeed the crack path.

Fig. 8 shows the result of the procedure describe before. The size of the spiral is particular impressive since we started with a small incision. It became that big only in a little bit more than two and a half turns. In principle this gave us a clue about which kind of this spiral it is.

3.3 Results: Measurement on the spiral

Because of the complicate way of the crack propagation, we cannot say "a priori" where is the center of the spiral. Also we cannot say that the crack propagation angle β is between the line that join the center of the spiral and the crack tip. In Fig 9 we show the geometrical way to find the center of the spiral and the parameters we are going to measure in the digitalized spiral.

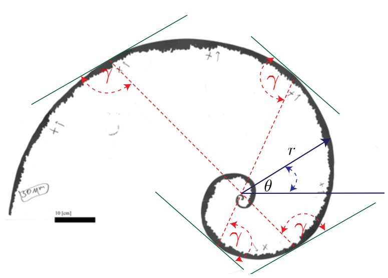

Figure 9: Over the spiral image we found a center by geometry

In Fig. 9 we show the property used to find a center in a logarithmic spiral. This property establish that the center must be somewhere between the line that join two points of parallel

tangents. Thus, we can find the center of the spiral with a pair of tangents.
The result of this measure for an spiral made in a 90 [μm] sheet is plotted in the following figure.

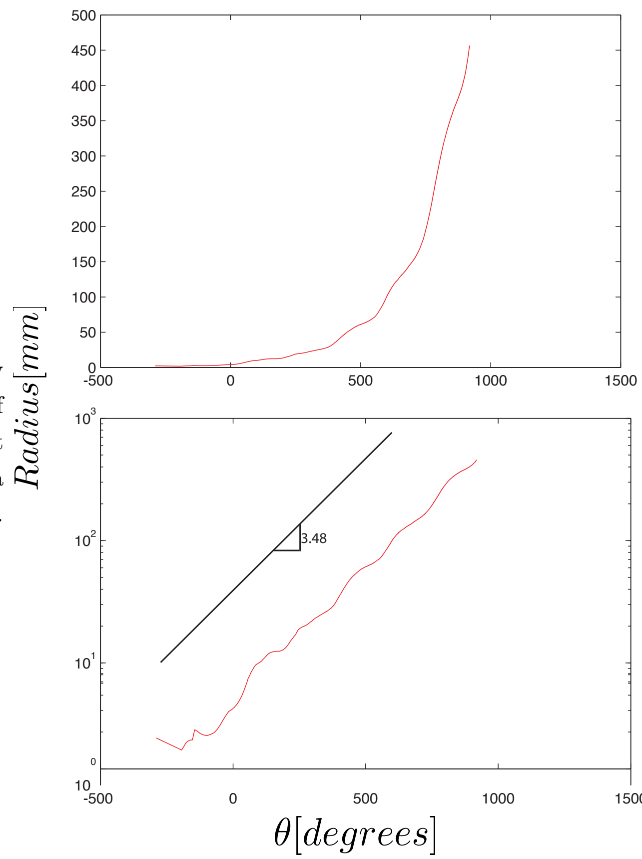

Figure 10: In this two plot we show the experimental result of one spiral of 90[μm]. The upper graphic is the plot of the measure. The lower graphic is a semi-log plot of the radius versus theta.

In order to understand these plots, we remark that these measurements were made from the center found in Fig. 9. This explains the behavior of the curve at the very beginning, because it represents the first two stages of the spiral respect to the center of the third step. These steps cannot have an exponential behavior respect to the center we are using. However, as it is patent in the lower plot in Fig. 10, the behavior of the radius is exponential over almost two decades. Despite this exponential behavior there are regular oscillation around the exponential line in the lower plot in Fig. 10.
These anomalies are obviously related with the orientation of the fracture process.The parameters that change with the orientation must be the ones associated with the anisotropy in the fracture process. In our case, this is the angle beta that defines the direction of propagation. In the next figure we show the angle β measured in each point of the spiral.

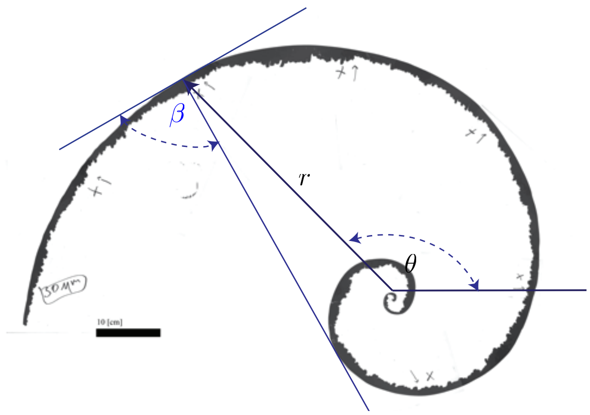

Figure 11: As we defined in the fracture process, the angle of the fracture propagation at each point is between the tangent to the spiral at this point and the line from the point to the outer point of the previous turn. This way we defined the angle β.

This angle β (the direction that guide the crack at each point) is plotted in the next figure against the angle θ to show its behavior with the orientation.

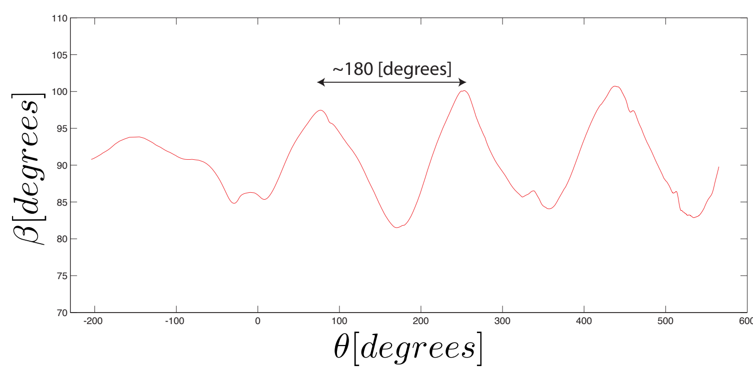

Figure 12: Plot of the angle β, a very constant 180 degrees periodicity is view in this plot.

In order to study this relation between the fracture process and the orientation of the film, we performed three different spirals. All the spirals was made in 90 [μm] sheets. Two of them started with the same orientation for the initial incision. For the third spiral we started with a 90 degrees rotated orientation for the initial incision. We show our results for three different spirals in the next figure.

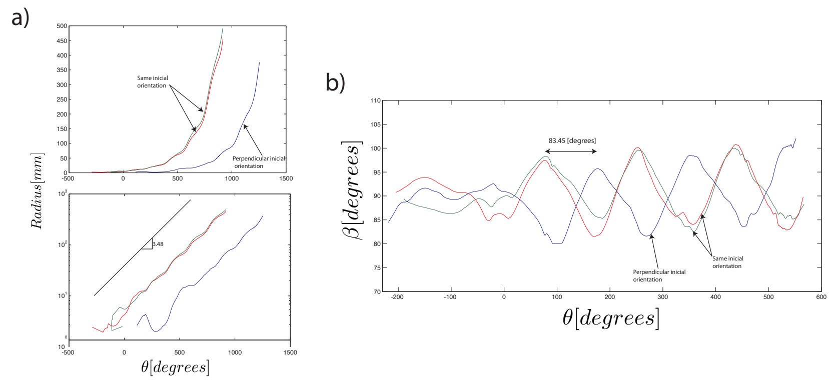

Figure 13: Three different spirals are plotted in the three graphics. **a**).- The three respective radius in a normal plot and in semi-log plot. **b**).- The plot of the angle β for each spiral.

It is particularly impressive the match between the two spirals in the plot **a**) in Fig. 13, also our guest is confirmed in plot **b**), in terms of the shift produced in the fracture angle β when the experiment is started in a different orientation.
One final experiment was performed. Here we measure the spiral parameters in spiraling cracks made in different thickness films of the same material. We show three different thickness, 30[μm] 50[μm] and 90 [μm], in the following figure.

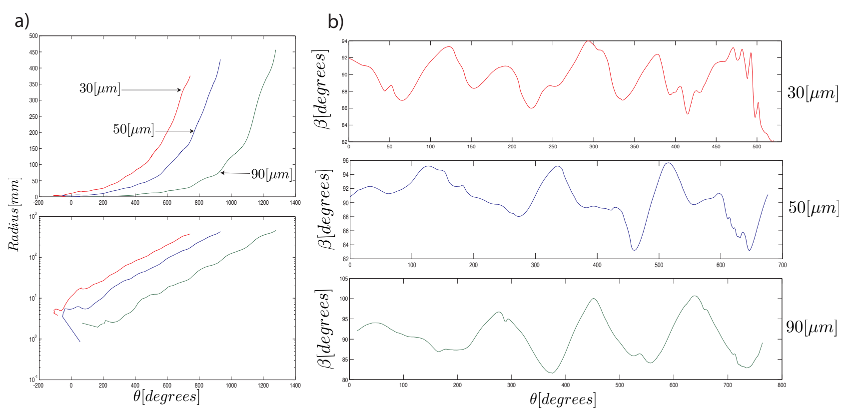

Figure 14: Three different spirals made in the same material but with different thickness are plotted in the three graphics. **a**).- The three respective radius in a normal plot and in semi-log plot. **b**).- The plot of the angle β for each spiral.

As we can see from the Fig. 14, the oscillations are persistent in each thickness. However the shape and the amplitude are different. In principle the thickness is not an important factor in fracture when the sheet is stretched and not bent, since stretching and fracture energy depend linearly with the thickness. However, the manufacturing process is not the same for films of different thickness, so that we can expect variations of the material properties for the three films.

3.4 Geometrical simulation of the fracture path

The main result of this work is that an explanation based on pure geometrical arguments can explain the observed spiral shape. However, we cannot expect that the oscillations presented previously are also implied by our model. To show that, we performed simulations of the fracture process following the geometrical rules of our geometrical model. These are: 1) to stress the lip until the angle α arise to its critical value, and 2) to propagate the crack in the direction that maximize the energy release rate.
In Fig. 15, we show a numerical spiral build using this procedure.

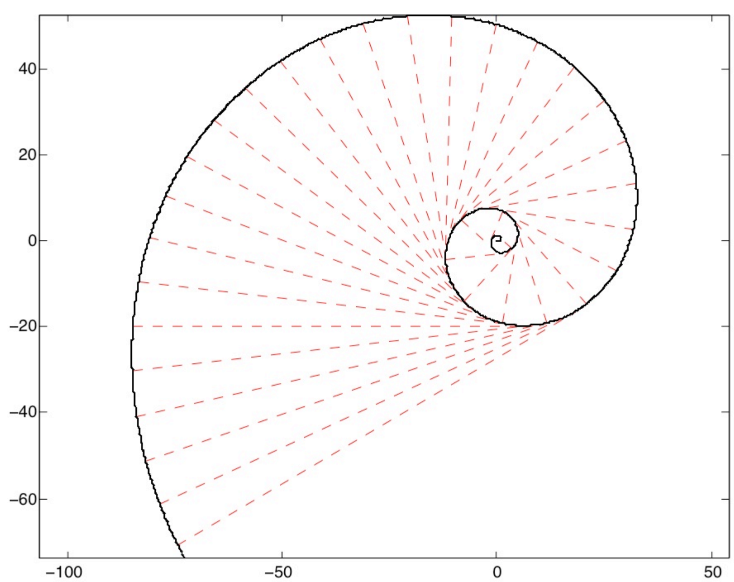

Figure 15: Plot of the simulated spiral. In this plot the red dashed lines shown the unstressed lips where the force was applied

Over this spiral we repeat the measurement procedure that we did with the experimental spirals. In Fig. 16, we show the plot between the distance to the center point and the angle. As you can see the behavior of this simulated spiral is complete logarithmic after some initial distance from the starting point.

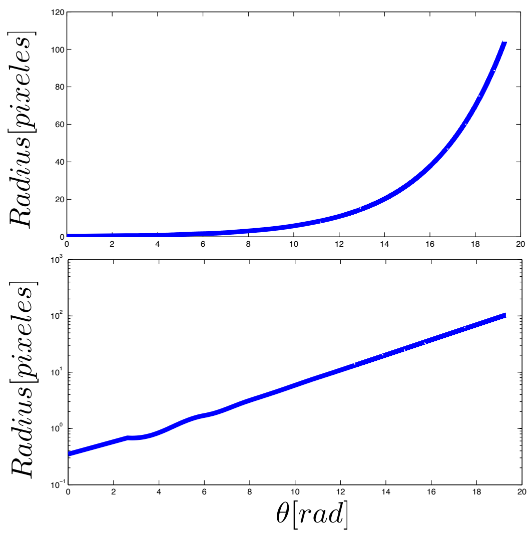

Figure 16: Plot of the measurement over the simulated spiral

These two figures confirm that the oscillations observed in a real fracture process are a phenomena connected with the anisotropy of the film material properties.

3.5 Anisotropy

It has been established that anisotropy affects the propagation of the crack in a thin sheet. As a matter of fact, it has been proved [**?**] that the criteria of local symmetry is no longer true when there is anisotropy in the material. As previously we pointed out, this criteria says that the crack will follow the path which makes zero the fracture shear mode (mode II). However, if there is anisotropy in the material, the propagation of the fracture not only will have a mode II component, but this mode will be a fundamental requirement for the propagation.
In our experiment, we observed a very robust connection between the crack propagation and the orientation. Despite this is not what we predict with our model, it is in some way intuitive because of the manufacturing process. To make the thin sheet that we used in our experiments, the manufacturing process stretches the material in two directions when it is still warm. Once they obtained the wanted thickness, they cool down very fast. This process introduces anisotropy in the structure and different material properties along two perpendicular directions. It could

also introduce a remanent stress. Thus, the oscillation observed in our spiral can be explained by these variations.

4 Conclusions

In this experiment we built up a setup which gives us a very robust crack path. The shape and the characteristics of the spiral we found are very reproducible. There is no need to big equipment to generate this spiral and the set up is quite simple.
Our simulations showed that the oscillations observed in the experiments are a consequence of the anisotropy in the material properties of our samples.
Despite the simpleness of the experiment it gives us important information about the fracture process and the material properties. The physics of the process is modeled by a simple and a very common criteria, Maximum of energy release rate. However this work will follow trying to modeling and understand the criteria that guides the fracture process in thin sheets.
Form this experiment we can see that the geometry is such a relevant factor when we are dealing with thin sheet, this is related with the freedom they have to change easily its shape. In this directions in this work we found a special rule for the grow of the spiral, this is a construction not around a center point but around it self.
For further work in this directions remains a more quantitative approach to the relation between the anisotropy and the fracture properties. Also the law for the critical angle of α have not be checked experimentally, it means that we do not know which parameter of the fracture process is affected by the anisotropy of the material.

5 Acknowledgements

This work have not been possible without the sponsorship of SCAT-alfa project. Also want to acknowledge the possibility to attend to the PMMH lab at the ESPCI is Paris to the professor Dr. Jose Eduardo Wesfreid director of the PMMH. Special acknowledge to the CNRS for the founding given to attend the APS March meeting 2008, where this work was presented in the Focus Session: Elasticity and Geometry of Thin Objects. To Conicyt and their Natiotal Scholarship for doctorate program. Finally to my advisors Dr Enrique Cerda and Benoit Roman, and despite is not an advisor professor, to Professor Dr. Jose Bico for his advices and the kindness of receive me and help me in the Paris life.

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